The Impact of Asset Allocation on Portfolio Recovery:
Quantifying the Value of Low Correlation

by

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Introduction

The process of building investment portfolios is facilitated by calculating the correlation between the assets under consideration. Building a portfolio using assets that have low correlation to each other is the standard approach. A well known benefit of building a low correlation portfolio is a reduction in the volatility of the portfolio returns (i.e. reduced standard deviation of return). This paper expands the investigation of the benefits of low asset correlation within a portfolio by examining (1) the impact of asset allocation on the ability of various retirement portfolios to recover from a loss, and (2) the impact of asset allocation on the historical likelihood of various retirement portfolios experiencing a loss.

Literature Review

The basic premise underlying diversification and portfolio selection can be summarized by a simple sentence by Harry Markowitz, "To reduce risk it is necessary to avoid a portfolio whose securities are all highly correlated with each other." (Markowitz, 1991). William Bernstein observes that "the concept of correlation of assets is central to portfolio theory—the lower the correlation, the better." (Bernstein, 2001). Geotzmann, Lingfeng and Rouwenhorst (2001) examined the correlation structure of the major equity markets over 150 years and found that "correlations vary considerably through time". Many have studied the correlation of markets [Bekaert and Harvey (1995); Bracker, Docking, and Koch (1999); Campbell and Hamao (1992); Erb, Harvye and Viskanta (1994); Grauer and Hakansson (1987); Grubel and Fadner (1971); Hamao, Masulis, and Ng (1990); Kaplanis (1988); Karolyi and Stulz (1996); Levy and Sarnat (1970); Longin and Solnik (1995); Longin and Solnik (2001)]. Recently, Coaker (2006) presented a pragmatic methodology for assessing the range of correlation between the S&P 500 Index and other prominent equity and bond asset classes.

Description and Justification

Correlation between two assets is measured in the range of -1.0 to +1.0, where negative 1.0 indicates that the price movement of two assets is perfectly inversely related. When one goes up, the other goes down, and vice versa. A correlation coefficient of zero indicates no correlation between the assets, while a correlation of 1.0 indicates perfect positive correlation. When one goes up, the other goes up. The magnitude of the positive or negative relationship is not measured by the correlation coefficient. That is the job of the slope, or beta, coefficient.

This study focuses on the aggregate correlation among various assets in a portfolio and the historical probability of recovery from a loss and the historical frequencies of losses sustained by the portfolio. Recovery is defined as restoring the portfolio account balance to its pre-loss balance. Moreover, this study specifically examines the impact of asset correlation in a retirement portfolio in withdrawal mode—or a portfolio in which an individual is withdrawing money on an annual basis. The withdrawal portfolio in this analysis assumes a starting balance of $500,000, an initial withdrawal at the end of the first year of 5% of the starting portfolio balance (in this case, $25,000), and an annual increase in the withdrawal of 3%. Thus, the second year withdrawal in this analysis was $25,750, the third year withdrawal was $26,523, and so forth.

Assets included in this study include large-cap US equities, small-cap US equities, non-US equities, US intermediate term bonds, commodities, real estate, and cash. The 36-year historical performance of large-cap US equities is represented by the S&P 500 Index, while the performance of small-cap US equities is captured by using the Ibbotson Small Companies Index from 1970-1978, and the Russell 2000 Index from 1979-2005. The performance of non-US equities was represented by the Morgan Stanley Capital International EAFE Index (Europe, Australasia, Far East) Index. U.S. intermediate term bonds were represented by the Ibbotson Intermediate Term Bond Index from 1970-76 and the Lehman Brothers Intermediate Term Bond index from 1977-2005. The historical performance of commodities was measured by the Goldman Sachs Commodities Index (GSCI). The performance of real estate investment trusts was measured by using the annual returns of the NAREIT Index (annual returns for 1970 and 1971 were estimated as the NAREIT Index did not provide annual returns until 1972). Finally, the historical performance of cash is represented by 3-month Treasury Bills. The annual returns of each asset from 1970-2005 are provided in Table 1. Data in parenthesis represent negative returns. The primary data source for this study was Morningstar Principia. Other data sources included Stocks, Bonds, Bills, Inflation by Ibbotson Associates.

In Table 2 the bivariate correlations between each asset are reported. For example, the36-year correlation between large US equity and small US equity was .738 or 73.8%. The correlation between US Intermediate Term bonds and commodities was -0.248. The aggregate correlation for the entire portfolio (as calculated by computing the average of the 21 bivariate correlations) was 0.132.

The data in upper portion of Table 3 report the needed returns to recover from a loss in a withdrawal portfolio (i.e., money is being systematically withdrawn). The lower half of Table 2 reports the needed returns to achieve recovery in a buy-and-hold portfolio in which no additional money is being invested and no money is being withdrawn. For example, to recover from a 10% loss, a withdrawal portfolio needs to generate an average annual return of 11.5% over the next 3 years (4 years if you count the initial "loss" year) or 9.4% per year over the next five years. Conversely, a buy-and-hold portfolio (in which there are no additional investments or withdrawals) needs only a 3.6% average annual return over the next three years to gain back a 10% loss (or 2.1% per year over five years).

As clearly seen by the required returns to restore a portfolio account balance following a loss, recovery is considerably more challenging in a withdrawal portfolio than in a buy-and-hold portfolio. In light of that, avoiding large losses in a withdrawal portfolio is the highest priority.

To calculate the probabilities of portfolio recovery from a loss and the frequency of losses within a portfolio, 36 years of historical performance were examined (from 1970-2005). Determining the likelihood of a portfolio recovering from a loss within 3 years was calculated by analyzing its historical performance over 34 unique three-year rolling periods. For example, the first three-year rolling period was 1970-1972, the second was from 1971-1973, and so on. Unlike Monte Carlo approaches, this approach does not simulate performance hundreds of thousands of times. Rather, this approach used rolling historical performance to create a set of historical three-year recovery probabilities as well as portfolio performance frequencies over one-year, two-year, and three-year periods.

Results

The first set of "portfolios" analyzed in this study included only one asset. As such they clearly do not represent a logical retirement portfolio, but are presented for comparison purposes. As shown in Table 4, the 36-year annualized return of the seven different single-asset "portfolios" ranged from 12.93% (REIT) to 6.37% (cash).

The 36-year annualized return of a 100% large US equity (S&P 500) portfolio was 11.06% with a standard deviation of return of 17.03%. In withdrawal mode, such a portfolio had a 64.7% probability of recovering from a 10% loss within 3 years. However, based on performance over the past 36 years, this same portfolio also had an 11.1% chance of experiencing a one-year loss of 10% or greater. Said differently, the 36-year historical frequency of a one-year return of -10% or greater was 11.1% in a 100% large US equity portfolio. The probability of a 100% large US equity portfolio having a cumulative loss of 10% or more over a two-year period was slightly lower (8.6%), but slightly higher over a three-year period (11.8%). (As can be seen, the loss frequencies tend to be highest over one-year periods, hence this study primarily focuses on the one-year performance frequencies).

A 100% bond portfolio had an 8.19% annualized return, 5.53% standard deviation, but only a 20.6% probability of recovering from a 10% loss. However, over the 36-year period of this analysis, a 100% bond portfolio never had a 10% cumulative loss (i.e., a 0% frequency) over a one-year, two-year, or three-year period. The bolded figures in Table 4 are displayed graphically in Figure 1. In general, among the five equity components (large US equity, small US equity, non-US equity, commodities, REIT), there is a negative correlation between probability of recovery and frequency of loss. Recall that the northwest quadrant (upper left) is the most desirable location in this type of graph inasmuch as it represents the combination of high probability of recovery and low frequency of loss.

We now turn to the analysis of retirement portfolios which incorporate multiple equally-weighted assets. As shown in Table 5, the first multi-asset portfolio is a two-asset portfolio consisting of 50% large US equity and 50% small US equity. The 36-year correlation of these two particular assets is .738. The annualized return of this portfolio was 11.75%, and had a standard deviation of return of 18.27%. The probability of recovering from a 10% loss within 3 years of the loss was 61.8%, and the frequency of a loss of 10% or more was 11.1%.

Next, non-US equities are added to the portfolio. Each of the three assets (large US equity, small US equity, and non-US equity) were equally weighted at 33.33% of the portfolio. The aggregate correlation of the portfolio falls from .738 to .598. The annualized return is 11.63%, or 12 bps lower than the two-asset portfolio, but the standard deviation of return drops by 89 bps. The recovery probabilities and frequencies of loss are comparable to the two-asset portfolio.

When adding bonds to the portfolio (now a four asset portfolio each asset comprising 25% of the portfolio) the return drops to 11.06%, but the volatility of return drops by 418 bps. The aggregate portfolio correlation drops considerably, from .598 to .334. The probability of recovery from a 10% loss remains the same as the three-asset portfolio (58.8%). However, after adding bonds to the portfolio, the frequency of a 10% loss drops dramatically from 13.9% to 8.3%. The aggregate correlation

Adding a cash component to the portfolio lowers aggregate correlation (to .234) and volatility of return in the portfolio (to 10.59%), but also reduces the

annualized return by 81 bps from 11.06% to 10.25%. Recovery probabilities are essentially unchanged, but the historical frequency of a 10% or greater loss drops to 2.8%.
The final two portfolios in Table 4 represent diversified asset mixes that fully exploit the impact of low correlation portfolios by adding commodities and real estate. The 36-year annualized performance of each portfolio is over 11%, comparable to the equity heavy single-asset and two-asset portfolios. The real impact of low correlation within a portfolio is observed by the dramatically reduced standard deviation of return (8.54% and 8.79% respectively). The probability of recovery is 55.9% for the six-asset portfolio and 52.9% for the seven-asset portfolio. Interestingly, the frequency of a 10% loss goes to zero in both the six and seven asset portfolio.

The final three portfolios included in this study are reported in Table 6. They are unequally-weighted, multiple-asset portfolios. The "Global Equity-Focused Portfolio" represents what might be considered a more aggressive retirement portfolio in comparison to the "Traditional Retirement Portfolio". It has an 80% commitment to equities (which includes 10% in commodities and 10% in REIT) compared to a 60% equity ratio in the traditional portfolio. Compared to the Traditional Retirement portfolio, the annualized return of the Global Equity-Focused portfolio is 157 bps higher, the aggregate correlation is lower (.167 to .234), the recovery probability is slightly higher, and the loss frequency is the same. The only downside is a slight increase in the standard deviation of return, from 10.61% to 11.17%. Only the most fastidious investor would be able to perceive such a small differential in performance volatility.

The 10 portfolios outlined in Tables 5 and 6 are plotted in Figure 2. It is instructive to observe their location in the graph. Eight of the ten multi-asset portfolios are clearly in the northwest quadrant. Not surprisingly, the two portfolios that are more to the right (pink dot = two-asset portfolio, yellow dot = three-asset portfolio) have higher aggregate portfolio correlation because fewer assets are being blended in the portfolio.

We have arrived at the heart of the issue of this study: What is the quantifiable value of low aggregate correlation of assets within a retirement portfolio in withdrawal mode? It is clear that reducing the aggregate correlation of assets within a portfolio has modest impact on performance, but can dramatically lower the standard deviation of return of the portfolio. Reducing volatility of return lowers the likelihood of large losses—which is the primary goal of a retirement withdrawal portfolio.

Compared to a 100% large US equity portfolio, a "traditional" retirement portfolio (Table 6) has a slightly lower probability of recovery from a loss (55.9% vs 61.8%). However, and this is the key point, recovery "power" (or probability) is not needed if the frequency of large loss is minimized, if not virtually eliminated. The 100% large US equity portfolio has an 11.1% chance of losing 10% in any one-year period, whereas the traditional retirement portfolio has only a 3% chance. By weighting equity assets more heavily in the Global Equity-Focused portfolio (and adding commodities and REITs to the portfolio) the probability of recovering from a 10% loss in a withdrawal portfolio increases to nearly 59%--very close to that of the 100% large US equity portfolio. More importantly, the frequency of loss (3%) was the same as the traditional retirement portfolio. The collective metrics (return, standard deviation, correlation, probability of recovery, frequency of loss) of the 10 multi-asset portfolios is presented in Figure 3.

Summary

The quantifiable value of low correlation among assets within a retirement withdrawal-mode portfolio is two-fold. First, there is a dramatic reduction in volatility of return. Second, there is a markedly reduced likelihood of loss in any given year. As shown by the mathematics of recovery in a retirement withdrawal portfolio (Table 3), avoiding large losses is of paramount importance. Achieving low correlation requires the use of a variety of assets, some of which may not fit the standard paradigm of a traditional withdrawal-mode retirement portfolio—namely commodities and/or REITs.

One word of caution is in order. The time frame of this analysis (1970-2005) was a period of robust returns across all asset classes. Equities averaged in excess of 10%, intermediate bonds averaged over 8%, commodities generated a 36-year average annual return of over 12%, and REITs had nearly a 13% average annual return. Going forward we cannot know if these returns will be replicable at these levels. It may be prudent to have more modest performance expectations over the next 7-10 years. Thus, even though the Global Equity-Focused portfolio demonstrated superior performance, low volatility of return, excellent recovery potential, and low frequency of losses—these findings are based on a period that favored such an asset mix.

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Table 1. Total Annual % Returns of Various Asset Classes & CPI

Year	Large US Equity	Small US Equity	Non-US Equity	Intermediate Term US Bonds	Commodities	REIT	Cash	All 7 Assets in Equal-Weighted Portfolio
1970	3.92	(17.40)	(11.66)	16.90	15.17	(4.00)	6.80	1.39
1971	14.14	16.50	29.59	8.70	20.15	15.52	4.52	15.59
1972	19.16	4.40	36.35	5.20	42.37	8.01	4.23	17.10
1973	(14.69)	(30.90)	(14.92)	4.60	74.90	(15.52)	7.46	1.56
1974	(26.47)	(19.90)	(23.16)	5.70	39.50	(21.42)	8.35	(5.34)
1975	37.23	52.80	35.39	7.80	(17.22)	19.29	6.08	20.20
1976	23.64	57.40	2.54	12.90	(11.94)	47.56	5.23	19.62
1977	(7.44)	25.40	18.06	3.00	10.38	22.43	5.52	11.05
1978	6.40	23.50	32.62	2.23	31.56	10.34	7.67	16.33
1979	18.30	43.07	4.75	6.59	33.78	35.86	10.86	21.89
1980	32.22	38.60	22.58	6.65	11.06	24.36	12.71	21.17
1981	(5.08)	2.03	(2.28)	10.79	(22.98)	6.02	15.58	0.58
1982	21.46	24.95	(1.86)	25.42	11.57	21.60	11.66	16.40
1983	22.46	29.13	23.69	8.22	16.23	30.64	9.24	19.94
1984	6.26	(7.30)	7.38	14.29	1.03	20.93	10.33	7.56
1985	31.74	31.05	56.16	18.00	10.02	19.07	7.97	24.86
1986	18.68	5.68	69.44	13.06	2.05	19.17	6.29	19.20
1987	5.26	(8.80)	24.63	3.61	23.76	(3.65)	6.13	7.28
1988	16.61	25.02	28.27	6.40	27.92	13.47	7.06	17.82
1989	31.68	16.26	10.54	12.68	38.25	8.84	8.67	18.13
1990	(3.12)	(19.48)	(23.45)	9.56	29.13	(15.34)	7.99	(2.10)
1991	30.48	46.04	12.13	14.11	(6.13)	35.69	5.68	19.71
1992	7.62	18.41	(12.17)	6.93	4.41	14.58	3.59	6.20
1993	10.06	18.88	32.56	8.17	(12.32)	19.67	3.12	11.45
1994	1.31	(1.82)	7.78	(1.75)	5.28	3.17	4.45	2.63
1995	37.53	28.45	11.21	14.41	20.32	15.25	5.79	18.99
1996	22.94	16.49	6.05	4.06	33.90	35.26	5.26	17.71
1997	33.35	22.36	1.78	7.72	(14.09)	20.28	5.31	10.96
1998	28.57	(2.55)	19.93	8.49	(35.71)	(17.51)	5.01	0.89
1999	21.04	21.26	27.03	0.49	40.89	(4.62)	4.87	15.85
2000	(9.10)	(3.02)	(14.19)	10.47	49.71	26.36	6.32	9.51
2001	(11.88)	2.49	(21.42)	8.42	(31.91)	13.93	3.67	(5.24)
2002	(22.09)	(20.48)	(15.94)	9.64	32.03	3.81	1.68	(1.62)
2003	28.67	47.25	38.59	2.29	20.68	37.14	1.05	25.10
2004	10.71	18.33	20.25	2.33	17.28	31.59	1.43	14.56
2005	4.91	4.55	13.54	1.68	25.69	12.17	3.34	9.41
36-Year Average Annualized Return	11.06	11.95	10.45	8.19	12.33	12.93	6.37	11.50
36-Year Standard Deviation of Return	17.03	22.13	22.02	5.53	23.87	16.53	3.14	8.79
Growth of $10,000 in Buy-and-Hold Portfolio	436,088	582,504	358,362	170,256	657,349	796,770	92,360	502,824

Table 2. 36-Year Correlations (1970-2005)

	Large US Equity	Small US Equity	Non-US Equity	US Bonds	Commodities	REIT
Small US Equity	0.738					
Non-US Equity	0.586	0.471				
US Bonds	0.228	0.070	-0.088			
Commodities	-0.275	-0.308	-0.126	-0.248		
REIT	0.466	0.771	0.293	0.143	-0.213	
Cash	0.042	0.004	-0.116	0.404	-0.005	-0.069
Aggregate (Average) Portfolio Correlation = 0.132						

Table 3: Needed Average Annual Return to Recover from Loss

Portfolio Return from which a Recovery is Needed	Needed **Average Annual** % Return to Restore Original Portfolio Balance				
	WITHDRAWAL Portfolio $500,000 initial balance, First Year Withdrawal of 5% of initial balance, 3% increase of annual withdrawal				
	Within 1 Year	Within 2 Years	Within 3 Years	Within 4 Years	Within 5 Years
5%	5.2%	5.2%	5.3%	5.4%	5.5%
2%	8.4%	6.9%	6.4%	6.3%	6.2%
0%	10.7%	8.0%	7.2%	6.9%	6.7%
-2%	13.1%	9.2%	8.0%	7.5%	7.2%
-5%	16.8%	11.1%	9.3%	8.4%	8.0%
-10%	23.7%	14.4%	11.5%	10.1%	9.4%
-15%	31.4%	18.0%	13.9%	12.0%	10.9%
-20%	40.2%	22.0%	16.5%	14.0%	12.5%
-25%	50.2%	26.4%	19.4%	16.1%	14.3%
-30%	61.8%	31.3%	22.6%	18.5%	16.2%
-35%	75.3%	36.9%	26.1%	21.2%	18.4%
Portfolio Return from which a Recovery is Needed	**BUY-and-HOLD Portfolio**				
	Within 1 Year	Within 2 Years	Within 3 Years	Within 4 Years	Within 5 Years
-2%	2.0%	1.0%	0.7%	0.5%	0.4%
-5%	5.3%	2.6%	1.7%	1.3%	1.0%
-10%	11.1%	5.4%	3.6%	2.7%	2.1%
-15%	17.6%	8.5%	5.6%	4.1%	3.3%
-20%	25.0%	11.8%	7.7%	5.7%	4.6%
-25%	33.3%	15.5%	10.1%	7.5%	5.9%
-30%	42.9%	19.5%	12.6%	9.3%	7.4%
-35%	53.8%	24.0%	15.4%	11.4%	9.0%

Table 4. Single-Asset "Portfolios" (1970-2005)

Single-Asset Withdrawal Portfolios **($500,000 starting balance, 5% withdraw rate, 3% inflation rate of withdrawal)**	36-Year Annualized % Return	36-Year Standard Deviation of Return (%)	% Probability of Recovery from a 10% Loss Within 3 Years	% Frequency of a **One-Year** Loss of 10% or more (1970-2005)	% Frequency of a **Two-Year** Cumulative Loss of 10% or more (1970-2005)	% Frequency of a **Three-Year** Cumulative Loss of 10% or more (1970-2005)
100% Large US Equity	11.06	17.03	**64.7**	**11.1**	8.6	11.8
100% Small US Equity	11.95	22.13	**52.9**	**13.9**	8.6	11.8
100% Non-US Equity	10.45	22.02	**41.2**	**22.2**	14.3	14.7
100% US Intermediate Bonds	8.19	5.53	**20.6**	**0.0**	0.0	0.0
100% Cash	6.37	3.14	**8.8**	**0.0**	0.0	0.0
100% Commodities	12.33	23.87	**44.1**	**19.4**	14.3	11.8
100% REIT	12.93	16.53	**55.9**	**11.1**	5.7	5.9

Table 5. Equally-Weighted Portfolios

Equally-Weighted Assets in Withdrawal Portfolio **($500,000 starting balance, 5% withdraw rate, 3% inflation rate of withdrawal)**	Annualized 36-Year Return (%)	36-Year Standard Deviation of Return (%)	Aggregate Portfolio Correlation	Probability of Recovery from a 10% Loss Within 3 Years (%)	Frequency of a One-Year Loss of 10% or more (%)
Two-Asset Large US Equity, Small US Equity (50% each)	11.75	18.27	.738	**61.8**	**11.1**
Three-Asset Large US Equity, Small US Equity, Non-US Equity (33.33% each)	11.63	17.38	.598	**58.8**	**13.9**
Four-Asset Large US Equity, Small US Equity Non-US Equity, US Int. Term Bonds (25% each)	11.06	13.20	.334	**58.8**	**8.3**
Five-Asset Large US Equity, Small US Equity, Non-US Equity, US Int. Term Bonds, Cash (20% each)	10.25	10.59	.234	**55.9**	**2.8**
Six-Asset Large US Equity, Small US Equity, Non-US Equity, US Int. Term Bonds, Cash, Commodities (16.67% each)	11.13	8.54	.092	**55.9**	**0.0**
Seven-Asset Large US Equity, Small US Equity, Non-US Equity, US Int. Term Bonds, Cash, Commodities, REIT (14.3% each)	11.50	8.79	.132	**52.9**	**0.0**

Table 6. Unequally-Weighted Portfolios

Unequally-Weighted Assets in Portfolio	Annualized 36-Year Return (%)	36-Year Standard Deviation of Return (%)	Aggregate Portfolio Correlation	Probability of Recovery from a 10% Loss Within 3 Years (%)	Frequency of a One-Year Loss of 10% or more (%)
Traditional Retirement Portfolio 50% Large US Equity, 5% Small US Equity, 5% Non-US Equity, 35% US Int. Term bonds, 5% Cash	10.26	10.61	0.234	**55.9**	**3.0**
Global Multi-Asset Portfolio 15% Large US Equity, 15% Small US Equity, 15% Non-US Equity, 35% US Int. Term bonds, 5% Cash 7.5% Commodities, 7.5% REIT	10.95	8.60	0.132	**47.1**	**0.0**
Global Equity-Focused Portfolio 20% Large US Equity, 20% Small US Equity, 20% Non-US Equity, 20% US Int. Term bonds, 0% Cash, 10% Commodities, 10% REIT	11.83	11.17	0.167	**58.8**	**3.0**

Figure 1. Single Asset “Portfolios”

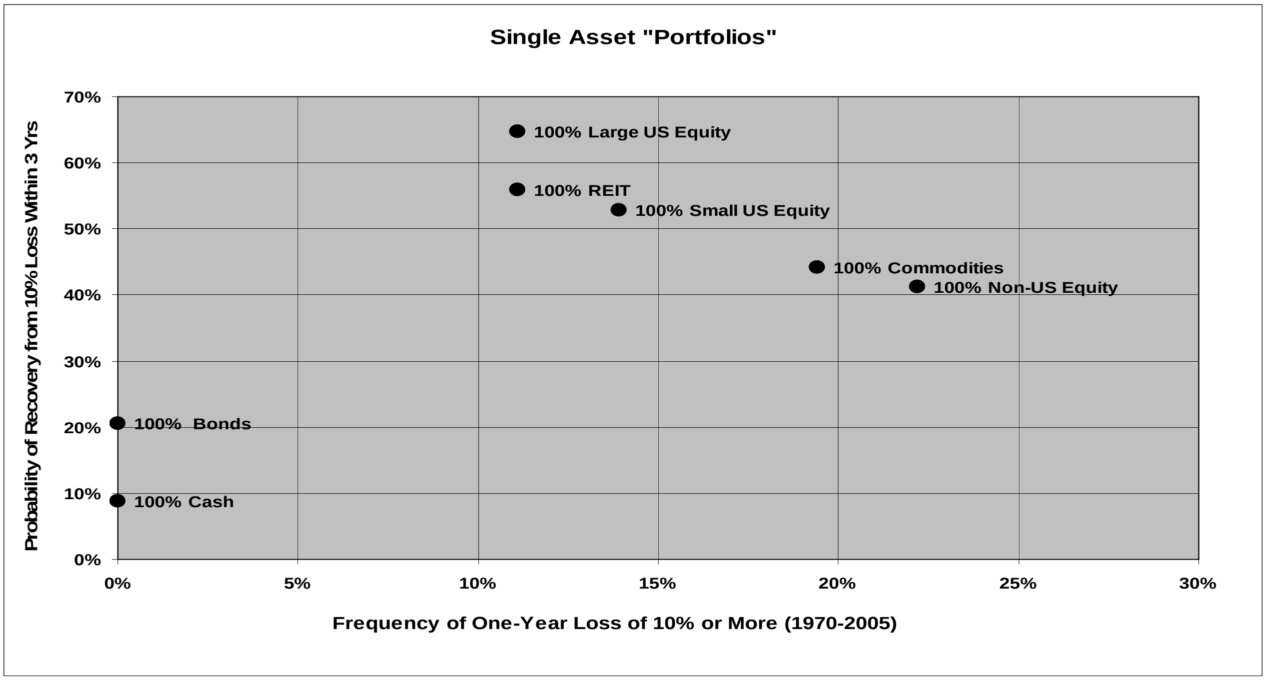

Figure 2. Multiple Asset Portfolios

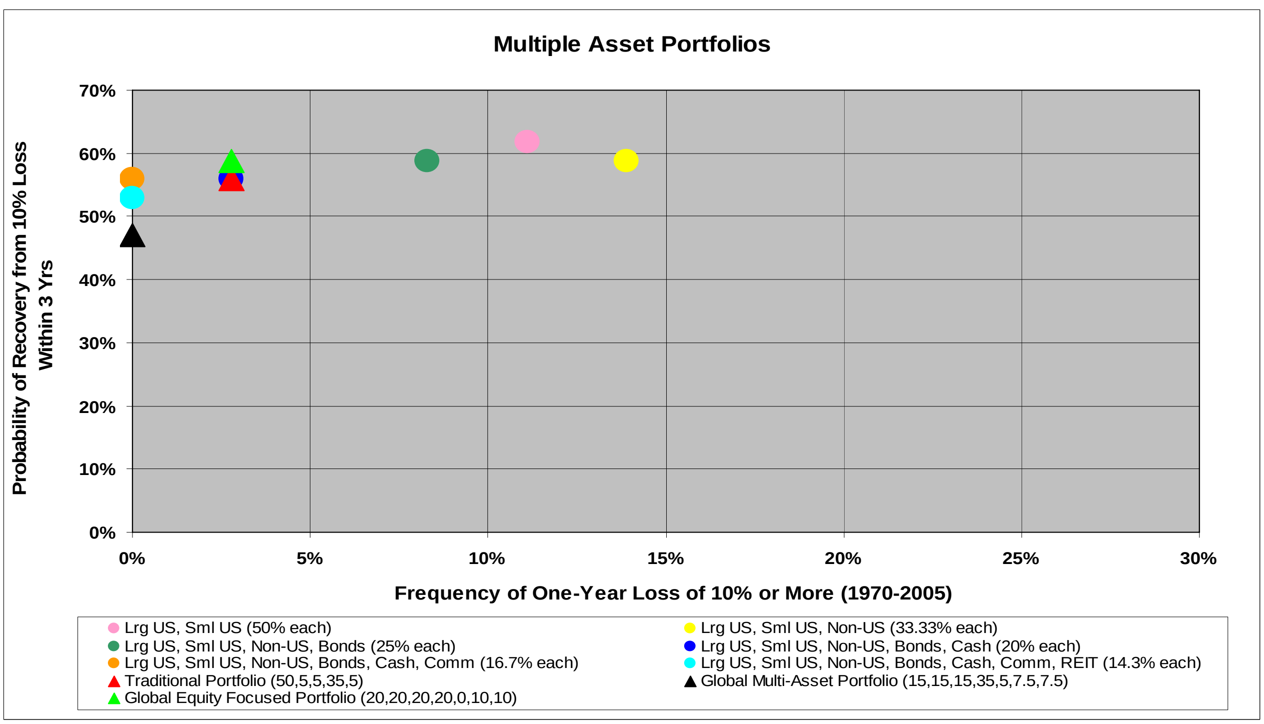

Figure 3. Probabilities of Recovery from a Loss of 10% in a Withdrawal Portfolio and Frequencies of Losses of 10% or Greater